Invariant distributions on the n-fold metaplectic covers of p-adic $GL(r, \mathbf{F})$ *

David Joyner

9-3-2000

Dedicated to John Benedetto on his 60^{th} birthday.

Abstract

We describe the unitary and tempered dual of the n-fold metaplectic covers of $SL(2, \mathbf{F})$, where $\mathbf{F}$ is a p-adic field with p not dividing $2n$. We show that any tempered distribution on the n-fold metaplectic covers of $SL(2, \mathbf{F})$ or of $GL(r, \mathbf{F})$ (satisfying the assumptions of §1.1 below) may be expressed as a distributional integral over the tempered dual. We also show that any invariant distribution on the n-fold metaplectic covers of $SL(2, \mathbf{F})$ or of $GL(r, \mathbf{F})$ is supported on the tempered dual.

1 Introduction

Since the days of Fourier, it has been known that any "nice" function on $\mathbb{R}$ has a Fourier transform, $f^\wedge(\pi) = \int_{\mathbb{R}} f(x)\pi(x)\, dx$, where $\pi \in \{e^{sx} \mid s \in \mathbb{C}\} = \mathbb{R}^\wedge$ belongs to the dual space. Let $(\mathbb{R})_u^\wedge$ denote the unitary dual space (π is unitary if and only if $s \in i\mathbb{R}$) and denote the Schwartz space by

$$\mathcal{S}(\mathbb{R}) = \left\{ f : \mathbb{R} \to \mathbb{C} \mid |D^n f(y)| \leq C \cdot (1 + |y|)^{-N} \right\},$$

*keywords: representation theory, metaplectic groups, harmonic analysis on p-adic reductive groups.
MSC: 22E50, 43A65, 11F70, 11F72.

where $C = C_{N,n,f} > 0$ is a constant. The image of the Schwartz space under unitary Fourier transform is $\mathcal{S}(\mathbb{R})^{\wedge}_u \cong \mathcal{S}(\mathbb{R})$. This result enables us to define, for each tempered $T \in \mathcal{S}(\mathbb{R})'$, $T^{\wedge} \in \mathcal{S}(\mathbb{R})'$ by $T^{\wedge}(f^{\wedge}) = T(f)$ (see for example, Benedetto [4], §1.3.6). A compactly supported tempered distribution is given by integration against some distributional derivative $D^n u$, some $n \geq 0$ and some $u \in C_c(\mathbb{R})$. These last few facts are well-known results of L. Schwartz [27]. Thanks, to R. Paley and N. Wiener, the image of $C_c^{\infty}(\mathbb{R})$ under unitary Fourier transform has also been classified (in terms of the "the Paley-Wiener space," a space of complex-analytic functions satisfying certain boundedness conditions).

We want analogs of these results for metaplectic covers of $SL(2, \mathbf{F})$ and $GL(r, \mathbf{F})$, where $\mathbf{F}$ is a p-adic field. In fact, Schwartz' classification of the compactly supported distributions will be used to prove its own p-adic analog. Moreover, the image of the Fourier transform shall be determined in the $SL(2)$ case.

If G is a connected reductive p-adic group with compact center then any (not necessarily tempered) invariant distribution on G is supported on the tempered dual ([22], [3]). In Theorem 4.6 below, we prove this result in the case of n-fold metaplectic covers $\overline{G}$ of $SL(2, \mathbf{F})$ or of $GL(r, \mathbf{F})$, as in §1.1 below. We also express the Fourier transform of any tempered distribution of $\overline{G}$ as a distributional integral over the tempered dual. Both of these results require some understanding of the tempered dual of $\overline{G}$.

A few words on possible applications.

First, Arthur invariant trace formula [2] is proving to be one of the most powerful and useful methods automorphic representation theory (which might be regarded as non-abelian harmonic analysis of reductive groups over the adeles $\mathbb{A}$ of a number field). Comparatively little has been done for non-linear groups, such as the metaplectic covering groups, due primarily to limitations in our knowledge of the non-abelian harmonic analysis of reductive groups over the p-adics. Theorem 4.5 is one of the assumptions needed to extend [2] to the metaplectic covers of $SL(2)$, $GL(r)$ (see [24] for the $GL(r)$ case). Thanks to results here, it appears that all the assumptions in local harmonic analysis have been proven now to establish the Arthur invariant trace formula on the 2-fold metaplectic cover of $SL(2, \mathbb{A})$. The trace formula for the 2-fold metaplectic cover of $SL(2, \mathbb{A})$ can be used to prove the multiplicity one conjecture for $SL(2)$ [21].

Second, one cannot help but notice the p-adic nature of the Walsh functions studied in wavelet theory [8]. Perhaps the (as far as I know undeveloped) theory of Walsh functions on non-abelian groups might find some relevancy in the ideas discussed here.

These results were presented at the Conference on Harmonic Analysis in honor of John Benedetto, University of Maryland, October, 1999.

1.1 Assumptions on $\overline{G}$

Throughout most of this paper we shall denote by G the group of $\mathbf{F}$-rational points of a connected reductive algebraic group $\underline{G}$ over $\mathbf{F}$. We denote by $\overline{G}$ a group which is a finite cyclic central topological extension,

$$1 \to \mu_n \to \overline{G} \to G \to 1,$$

where μ_n denotes the group of n^{th} roots of unity, $\mathbf{F}$ contains all n^{th} roots of unity,

- $\underline{G} = SL(2)$ and p does not divide $2n$,

or

- $\underline{G} = GL(r)$, p does not divide n, and n is relatively prime to all composite positive integers less than or equal to r.

We denote the above projection by $\rho : \overline{G} \to G$.

Let $\mathbf{F}$ be a p-adic field with uniformizer $\pi_{\mathbf{F}}$, ring of integers $\mathcal{O}_{\mathbf{F}}$, residual characteristic $p = char(\mathcal{O}_{\mathbf{F}}/\pi_{\mathbf{F}}\mathcal{O}_{\mathbf{F}})$, $q = |\mathcal{O}_{\mathbf{F}}/\pi\mathcal{O}_{\mathbf{F}}|$, and normalized valuation $|...|_{\mathbf{F}}$. Let

$$N = \begin{cases} n, & n \text{ odd}, \\ n/2, & n \text{ even} \end{cases} \tag{1}$$

and let $\underline{N_0}$ denote the unipotent upper triangular subgroup of $\underline{G}$.

For $g = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in G = SL(2, \mathbf{F})$, let

$$x(g) = \begin{cases} c, & c \neq 0, \\ d, & c = 0, \end{cases}$$

and let $\beta = \beta_{n,\mathbf{F}} : G \times G \to \mu_n$ be defined by

$$\begin{aligned} \beta(g_1, g_2) &= (x(g_1), x(g_2))_n \cdot (-x(g_1)^{-1}x(g_2), x(g_1g_2))_n \\ &= (\frac{x(g_1g_2)}{x(g_1)}, \frac{x(g_1g_2)}{x(g_2)})_n, \end{aligned} \tag{2}$$

where $(..., ...)_n = (..., ...)_{n,\mathbf{F}} : \mathbf{F}^\times \times \mathbf{F}^\times \to \mu_n$ denotes the Hilbert symbol [31]. This cocycle defines a cover $\overline{G}$ satisfying the properties above. Elements of $\overline{G}$ will be denoted by (g, ς), where $g \in G$, $\varsigma \in \mu_n$.

The cocycle for $GL(r, \mathbf{F})$ is described in [9].

1.2 Basic notation

If H is any subset of G then denote $\overline{H} = \rho^{-1}(H)$. In particular, if G_r denotes the set of regular elements of G in the sense of [22], let $\overline{G}_r$ be the pull-back of G_r via the projection ρ.

Let $\mathcal{L}(G)$ denote the set of standard Levi subgroups of G (with respect to a given maximal split torus of G). We write A for the diagonal subgroup of G. Let $\mathcal{L}(\overline{G})$ denote the set of Levis in $\mathcal{L}(G)$ pulled back to $\overline{G}$ via ρ. We call these the standard Levi subgroups of $\overline{G}$. For each $M \in \mathcal{L}(G)$, let $X(M)$ denote the variety of unramified characters of M and let $X^{un}(M)$ denote the variety of unramified unitary characters of M. If $M = \overline{A} \in \mathcal{L}(\overline{G})$, let $X(M)$ denote the variety of unramified characters of A^n (which we may identify with a character of $\overline{A^n}$) and let $X^{un}(M)$ denote the variety of unramified unitary characters of A^n. Let $W = N_G(A)/A$ denote the Weyl group of A. When $G = SL(2, \mathbf{F})$, we sometimes identify W (as a set) with $\{1, w_0\}$ or sometimes (as a group) with $\{1, w_1\}$, where $w_0 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \in G, w_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, and when W is to act on $\overline{A}$ instead of A, we sometimes identify (using a slight abuse of notation) W (as a set) with $\{1, \overline{w_0}\}$, where $\overline{w_0} = (w_0, 1)$.

We shall often identify the unipotent radical $N = \{\begin{bmatrix} 1 & & * \\ & \ddots & \\ 0 & ... & 1 \end{bmatrix}\} \subset G$ with the subgroup $\{(n, 1) \mid n \in N\} \subset \overline{G}$. Let $K_0 = \underline{G}(\mathcal{O}_{\mathbf{F}})$. It is known that if $(p, 2n) = 1$ then $\overline{K_0}$ splits (see [10] for $SL(2)$ and [9] for $GL(r)$).

We call a function f of $\overline{G}$ (resp., of any subgroup H of $\overline{G}$) *genuine* if it satisfies

$$f(g, \varsigma) = \varsigma^{-1} \cdot f(g, 1), \tag{3}$$

for all $g \in G$ (resp., $(g, \varsigma) \in H$). Let $C_c^\infty(G)$ denote the space of smooth (i.e., locally constant and compactly supported) functions on G and let $C_c^\infty(\overline{G})$ denote the space of smooth genuine functions on $\overline{G}$.

Let $||g|| = \max(|g_{ij}|)$, where $g = [g_{ij}] \in G$, and let $\sigma(g) = \log ||g||$. For each compact open subgroup $K \subset\subset \overline{G}$, let

$$\mathcal{S}_K(\overline{G}) = \left\{ \begin{array}{c} f \in C_c(\overline{G}//K) \mid f \text{ genuine}, \\ |f(x)| \leq C \cdot \frac{\Xi(x)}{(1+\sigma(x))^r}, \quad \forall \overline{x} = (x, \varsigma) \in \overline{G}, \\ \text{for each } r > 0 \end{array} \right\},$$

where $C = C_{r,f} > 0$ is a constant, where $C_c(\overline{G}//K)$ denotes the space of compactly supported functions which are bi-K-invariant and where $\Xi(x) = \int_{K_0} \delta_B(xk)^{-1/2} dk$. Here, δ_B denotes the usual modulus function defined for $\overline{x} = (x, \zeta) \in \overline{G_r}$ by

$\delta_B(\overline{x}) = |\det(Ad(x_d))_{\mathfrak{n}}|$, where x_d denotes a diagonalization of x in $\underline{G}(\overline{\mathbf{F}})$, where $\overline{\mathbf{F}}$ denotes a separable algebraic closure of $\mathbf{F}$ and the valuation $|...|$ has been extended to $\overline{\mathbf{F}}$, and where $\mathfrak{n}$ denotes the Lie algebra of the unipotent upper triangular N (more precisely, the Lie algebra of $\underline{N}(\overline{\mathbf{F}})$). We topologize $\mathcal{S}_K(\overline{G})$ via the semi-norms $\upsilon_k(f) = \sup_{x\in\overline{G}} |f(x)| \frac{(1+\sigma(x))^k}{\Xi(x)}$. Let $\mathcal{S}(\overline{G}) = \bigcup_K \mathcal{S}_K(\overline{G})$, where K runs over all compact open subgroups of $\overline{G}$. This is the (p-adic [1]) *Schwartz space* of $\overline{G}$. Let S denote the collection of all seminorms on $\mathcal{S}(\overline{G})$ whose restriction to each $\mathcal{S}_K(\overline{G})$ is continuous. In the semi-norm topology induced by S, the Schwartz space is a complete locally convex topological vector space. Moreover, $\mathcal{S}(\overline{G}) \subset L^2(\overline{G})$ and $\mathcal{S}(\overline{G})$ is an algebra under convolution.

We call a representation π of $\overline{G}$ (resp., of any subgroup H of $\overline{G}$) *genuine* if it satisfies

$$\pi(g, \varsigma) = \varsigma \cdot \pi(g, 1), \tag{4}$$

for all $g \in G$ (resp., $g \in \rho(H)$). If π denotes an admissible representation of G then let Θ_π denote the character of π. Likewise, if π denotes an admissible genuine representation of $\overline{G}$ then let Θ_π denote the character of π (see [16], §4, for details on how to extend the results in §4.8 of [28] to metaplectic covers). We may regard Θ_π as either a locally integrable genuine function on $\overline{G}_r$, or as an invariant distribution, whichever is appropriate for the context, as in [12], [13].

For each Levi component M of $\overline{G}$, let

- $\Pi(M) = \Pi(M)_a$ denote the genuine admissible dual of M (the set of equivalence classes of genuine irreducible admissible representations[2] of M),
- $\Pi(M)_t$ denote the genuine tempered dual of M,
- $\Pi(M)_u$ denote the genuine unitary dual of M,

Furthermore, let

- $R_t(\overline{G})_{\mathbb{Z}}$ denote the Grothendieck group of genuine, tempered, admissible representations of $\overline{G}$ and let $R_t(\overline{G}) = R_t(\overline{G})_{\mathbb{Z}} \otimes_{\mathbb{Z}} \mathbb{C}$,
- $PW_t(\overline{G})$ denote the *tempered Paley-Wiener space*, i.e., the space of functionals on $R_t(\overline{G})$ of the form $\phi_f : \pi \to \Theta_\pi(f)$, for some $f \in \mathcal{S}(\overline{G})$. Similarly, let $PW(\overline{G})$ denote the *Paley-Wiener space*, i.e., the space of functionals on $R(\overline{G})$ of the form $\phi_f : \pi \to \Theta_\pi(f)$, for some $f \in C_c^\infty(\overline{G})$.

[1] In the p-adic case, the letter $\mathcal{C}$ is often used for the Schwartz space, rather than the letter $\mathcal{S}$.

[2] From this point on, all representations will be assumed to be admissible unless otherwise stated.

2 Basic lemmas on orbital integrals

If $p < \infty$ then $\mathbf{F}^\times = \pi^\mathbb{Z} \cdot \mu_{q-1} \cdot U_1$, a direct product. If $(p, 2n) = 1$ then $\mu_n \subset \mathbf{F}^\times$ implies $q \equiv 1 \pmod{n}$. Recall N is defined in (1).

Lemma 2.1. *Suppose A is the diagonal subgroup of $SL(2, \mathbf{F})$.*

(a) If $(p, N) = 1$ then $C = \pi^\mathbb{Z} \mathcal{O}_\mathbf{F}^{\times N} = \pi^\mathbb{Z} \mu_{q-1}^N (1 + \pi\mathcal{O}_\mathbf{F})$ is a maximal subgroup of $\mathbf{F}^\times$ for which $\overline{h(C)} \subset \overline{A}$ is abelian, where $h(x) = diag(x, x^{-1})$.

(b) If $(p, n) = 1$ then $C = \pi^\mathbb{Z}(1 + \pi\mathcal{O}_\mathbf{F})\mu_{q-1}^N$ has index N in $\mathbf{F}^\times$.

The (straightforward) verification of this fact is omitted (see [16] or [18]).

Let $D(h(a)) = D_{G/A}(\begin{bmatrix} a & 0 \\ 0 & a^{-1} \end{bmatrix}) = \det(1 - Ad(\begin{bmatrix} a & 0 \\ 0 & a^{-1} \end{bmatrix}))_{\mathfrak{g}/\mathfrak{a}} = -(a - a^{-1})^2$, where $\mathfrak{g}, \mathfrak{a}$ denote the Lie algebras of G, A, resp.. This map pulls back to $\overline{G}_r$ via ρ. For $t \in \overline{G}_r$, let $T = Cent(t, \overline{G})$, denote the centralizer[3]. Define the *orbital integral* of $f \in C_c^\infty(\overline{G})$ by

$$F_f^T(t) = |D(t)|^{1/2} \int_{T\backslash\overline{G}} f(x^{-1}tx) \frac{dx}{dt}. \tag{5}$$

(This exists as a simple consequence of a well-known result of Harish-Chandra [12]. We define D as above by identifying T with A over the algebraic closure.) If $a \in \overline{A}$ is regular, then define

$$F_f^{A^N}(a) = |D(a)|^{1/2} \int_{\overline{A^N}\backslash\overline{G}} f(x^{-1}ax) \frac{dx}{da}. \tag{6}$$

Lemma 2.2. *Let $\overline{G}$ be a cover of $SL(2, \mathbf{F})$ as in §1.1.*

(a) For $f \in C_c^\infty(\overline{G})$, $a \in \overline{A} - \overline{A^N}$, we have $F_f^{A^N}(a) = 0$.

(b) The map $f \longmapsto F_f^{A^N}$ defines a surjection $C_c^\infty(\overline{G}) \to C_c^\infty(\overline{A^N})^W$, where the action of W on $\overline{A^N}$ is as in §2.

(c) The map $f \longmapsto F_f^{A^N}$ defines a continuous surjection $\mathcal{S}(\overline{G}) \to \mathcal{S}(\overline{A^N})^W$.

The relatively straightforward proofs of these results will be omitted. (However, one may find detailed proofs in [16].)

For orbital integrals on $GL(r, \mathbf{F})$, we refer to chapter 1 of [9]. For example, the analog of part (b) above follows from §I.7-I.8 in [9].

[3] Note T need not be equal to the metaplectic cover of a centralizer of G. In other words, if $t = (x, 1)$ then in general $Cent(t, \overline{G}) \neq \overline{Cent(x, G)}$.

3 Unitary and tempered dual of $\overline{SL(2, \mathbf{F})}$

In this section, let $\overline{G}$ be a cover of $SL(2, \mathbf{F})$ as in §1.1. We shall need some facts about the tempered dual of $\overline{G}$ for the main result in the next section. In particular, we recall the classification of the unitary and tempered dual of $\overline{G}$ in order to state a theorem of "Paley-Wiener type" for the unitary and tempered Fourier transforms in §4.1 below. All the results of this section are essentially in the literature in one form or another but see [18], [20] for more details.

For $\overline{GL(r, \mathbf{F})}$, the necessary results on the tempered dual may be deduced from §19 of [9].

It is remarked in [5], §2.2 that the arguments of [6], chapter 2 carry over to finite central extensions of reductive groups over a p-adic field (see also [23], §1.2). The arguments of [7], section 2, also carry over to finite central extensions of split reductive groups over a p-adic field. Such results reduce the determination of the unitary dual of $\overline{G}$ down to classifying the supercuspidal representations (done in [19] when $gcd(p, n) = 1$ and [17] for any p, n) and the constituents of the induced representations.

3.1 Principal series

If $\overline{P} = \overline{M}N$ denotes a Levi decomposition of a parabolic subgroup of $\overline{G}$ and (σ, W) is a genuine supercuspidal representation of $\overline{M}$ (which we extend to $\overline{P} = \overline{M}N$ trivially), then define $I_M(\sigma) : \overline{G} \to Aut(V)$ to be the *unitarily induced representation*: the representation of $\overline{G}$ by right translation on

$$V = \left\{ f : \overline{G} \to W \ \text{ genuine} \mid \begin{array}{c} (1) \;\; f(mg) = \delta_P(m)^{1/2}\sigma(m)f(g), \\ \forall g \in \overline{G}, \; m \in \overline{M} \\ (2) \;\; \text{for some compact open subgp } K \subset \overline{G}, \\ f(gk) = f(g), \;\; \forall k \in K, \; g \in \overline{G} \end{array} \right\}.$$

Here δ_P denotes the modulus function in §1.2.

In general, if ρ is a representation of a group $H \subset G$ and $x \in N_G(H)$ then we let ρ^x be the representation defined by $\rho^x(h) = \rho(x^{-1}hx)$, for $h \in H$.

Let $\chi, \chi' \in \Pi(\overline{A})$ and let $\overline{w} = (w, 1)$, for $w \in W$. If $\chi^{\overline{w}} \neq \chi$ for all $w \in W - \{1\}$ then we call χ *regular*. We say that χ, χ' are W*-conjugate* if $\chi' = \chi^{\overline{w}}$ for some $w \in W$. It is known that distinct W-conjugacy classes of $\chi \in \Pi(\overline{A})$ yield inequivalent representations.

Suppose that $\pi \in \Pi(\overline{G})_u$. We call π a (unitary) *principal series* representation if $\pi = I_{\overline{A}}(\chi)$ for some $\chi \in \Pi(\overline{A})_u$. These representations are tempered. In case $I_{\overline{A}}(\chi)$ is reducible and $\chi \in \Pi(\overline{A})_u$, we call the irreducible constituents *reducible principal series* (or, more precisely, *reducible principal series constituents*).

Let $\chi \in \Pi(\overline{A})$. The induced representation $I_{\overline{A}}(\chi)$ is in general not irreducible. However, we do have the following result.

Proposition 3.1. *(Moen [26])*

(a) If n is even and $gcd(p, n) = 1$ then $I_{\overline{A}}(\chi)$ is irreducible and unitary for all $\chi \in \Pi(\overline{A})_u$.

(b) If n is odd and $gcd(p, n) = 1$ then $I_{\overline{A}}(\chi)$ is irreducible and unitary for all $\chi \in \Pi(\overline{A})_u$ such that (a) $\chi = 1$ or (b) $\chi^{\overline{w_0}} \neq \chi$ where $\overline{w_0} = (w_0, 1)$. If $\chi^{\overline{w_0}} = \chi$ and $\chi \neq 1$ then $I_{\overline{A}}(\chi)$ is reducible and has two irreducible constituents.

In fact, C. Moen [25] explicitly computes the intertwining operators as matrices using the Kirillov model when $gcd(p, n) = 1$.

Proposition 3.2. *$I_{\overline{A}}(\chi)$ is irreducible and unitary for all $\chi \in \Pi(\overline{A})_u$ such that $\chi^{\overline{w_0}} \neq \chi$ where $\overline{w_0} = (w_0, 1)$.*

The above result has a direct proof, based on Bruhat theory, but it can also be deduced from results in [9].

3.2 Complementary series

In this subsection, we shall briefly review some of the results of Ariturk [1] and use some results of Flicker and Kazhdan [9] to generalize them to the n-fold cover ([1] assumed $n = 3$ and $p > 3$). In case $n = 2$, these results were essentially known to Gelbart-Sally [11].

We call an irreducible unitary representation π a *complementary series* representation if $\pi = I_{\overline{A}}(\chi)$ for some $\chi \in \Pi(\overline{A}) - \Pi(\overline{A})_u$. These representations are not tempered.

Let $C \subset A$ be a maximal subgroup of A for which $\overline{C} \subset \overline{A}$ is abelian. Let $\mu \in \Pi(\overline{C})$, $\chi = \chi_\mu = Ind_{\overline{C}}^{\overline{A}}\mu \in \Pi(\overline{A})$. If $\mu(x) = \mu_0(x)|x|^s$, for some character μ_0 of finite order and some $s \in \mathbb{C}$ then we write $s = s(\mu) = s(\chi)$.

Let $K(\mu)$ denote the space of locally constant functions $f : F \times \overline{A} \to \mathbb{C}$ such that

(i) $f(x, a_1a_2) = \mu(a_1)f(x, a_2)$, $a_1 \in \overline{C}$, $a_2 \in \overline{A}$,

(ii) $|x|\chi(\begin{bmatrix} x & 0 \\ 0 & x^{-1} \end{bmatrix}, 1)f(x, a)$ is constant for $|x|$ large.

Let $R \subset \overline{A}$ denote a complete set of representatives of $\overline{A}/\overline{C}$, and let r denote the cardinality of R. The elements $f \in K(\mu)$ may be identified with the r-tuple $(f(x, a))_{a \in R}$.

Let $V(\mu)$ denote the space of all locally constant functions $\varphi : \overline{G} \times \overline{A} \to \mathbb{C}$ such that

(i) $\varphi(g, a_1 a_2) = \mu(a_1)\varphi(g, a_2)$, $a_1 \in \overline{C}$, $a_2 \in \overline{A}$,

(ii) $\varphi(a_1 n g, a_2) = \delta(a_1)\varphi(g, a_2 a_1)$, where $a_1 \in \overline{A}$, $a_2 \in \overline{A}$, $n \in N_0$. Here δ denotes the usual modulus function as defined in §1.2 above. For $\varphi \in V(\mu)$ and $w \in W$, define the map $T = T_w$ by

$$T\varphi(g, a) = \int_F \varphi(\overline{w} \cdot (\begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix}, 1) \cdot g, \overline{w} a \overline{w}^{-1}) dx, \quad Re(s(\mu)) > 0,$$

where $\overline{w} = (w, 1)$.

Lemma 3.3. *(Ariturk) T intertwines $I_{\overline{A}}(\mu)$ and $I_{\overline{A}}(\mu^{\overline{w}})$.*

This lemma does not require us to assume $gcd(p, n) = 1$.

Let $L(\overline{G}, \overline{B})$ denote the space of all locally constant functions φ on $\overline{G}$ such that

$$\varphi((\begin{bmatrix} a & * \\ 0 & a^{-1} \end{bmatrix}, \varsigma) \cdot g) = |a|^2 \varphi(g).$$

For $\varphi_1 \in V(\mu)$, $\varphi_2 \in V(\mu^w)$, the function $g \longmapsto \int_{\overline{A}/\overline{C}} \varphi_1(g, a)\varphi_2(g, a) da$ belongs to $L(\overline{G}, \overline{B})$. Therefore, $\langle \varphi_1, \varphi_2 \rangle = \int_{\overline{B}\backslash\overline{G}} \int_{\overline{A}/\overline{C}} \varphi_1(g, a)\varphi_2(g, a) da dg$ gives a non-degenerate bilinear form on $V(\mu) \times V(\mu^{\overline{w}})$.

Lemma 3.4. *(Ariturk) $I_{\overline{A}}(\mu^{\overline{w}})$ is the contragredient of $I_{\overline{A}}(\mu)$.*

This lemma does not require us to assume $gcd(p, n) = 1$.

For $f \in K(\mu)$, define the Fourier transform of f by $f^\wedge(x, a) = \int_{\mathbf{F}^\times} f(y, a)\psi(xy) dy$, where ψ is a fixed additive character of $\mathbf{F}$.

Lemma 3.5. *(Ariturk) Assume $gcd(p, n) = 1$. For φ_1, $\varphi_2 \in V(\mu)$, $\mu(x) = |x|^s$, we have*

$$\langle \varphi_1, T\varphi_2 \rangle = \int_{\mathbf{F}} \int_{\overline{A}/\overline{C}} f_1^\wedge(x, a) \overline{(J f_2^\wedge)(x, a)} da dx,$$

where $J = J_\mu$ is a linear transformation on $K(\mu)^\wedge$ and $f_i(x, a) = \varphi_i(\overline{w}^{-1} \cdot \begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix}, a)$, $i = 1, 2$.

We may identify the map $J = J_\mu$ defined in the above lemma with an $r \times r$ matrix which we still denote by J.

Lemma 3.6. *(Langlands, Ariturk) Assume $gcd(p, 2n) = 1$. If $0 \leq Re(s(\mu)) \leq 1/n$ and $|Im(s(\mu))| \leq \pi/n \ln(q)$ then the image of J_μ is an irreducible representation of $\overline{G}$.*

Proposition 3.7. *(Flicker-Kazhdan) If* $0 < s(\mu) < 1/n$ *then* $I_{\overline{A}}(\mu)$ *is a unitarizable representation of* $\overline{G}$.

Corollary 3.8. *If* $0 < s < 1/n$ *and* $\mu(x) = |x|^s$ *then* $\langle \varphi_1, T\varphi_2 \rangle$ *is a positive definite form. In particular,* $I_{\overline{A}}(\mu)$ *is unitary in this range. If, in addition,* $gcd(p, 2n) = 1$ *then* J_μ *is a positive definite matrix.*

3.3 The special and the "trash" representations

For the origin of the term "trash" representation, see [10].

As a consequence of the above-mentioned facts, we have the following result.

Proposition 3.9. *Let* $s = 1/n$ *and* $\mu(x) = |x|^s$.

(a) *The irreducible subrepresentation of* $I_{\overline{A}}(\mu)$ *(if* $gcd(p, 2n) = 1$*, the kernel of* J_μ*) is the "special" representation* π_{sp}. *It is tempered and square-integrable (hence unitary). If* $gcd(p, n) = 1$ *then it also contains an Iwahori fixed vector.*

(b) *If* $n > 1$ *then the irreducible quotient of* $I_{\overline{A}}(\mu)$ *(if* $gcd(p, 2n) = 1$*, image of* J_μ*) is an infinite-dimensional, non-tempered representation* π_{nt}. *If* $gcd(p, n) = 1$ *then it is also spherical.*

Remark 3.10. *In the case* $n = 3$ *and* $p > 3$*, this proposition follows from [1]. In case* $n = 2$*, most of the statements are proven in [11].*

Proposition 3.11. *(Kazhdan-Patterson [23])* [4] π_{nt} *is unitary.*

Remark 3.12. *This was known earlier in the cases* $n = 2$ *([11], Theorem 2) and* $n = 3$, $p \neq 3$ *([1], Theorem 5.4).*

3.4 Classification

We summarize the above results.

Theorem 3.13. *Let* $\overline{G}$ *be as in §1.1 above. If* $\pi \in \Pi(\overline{G})_u$ *then one of the following holds.*

- *(Principal series) There is a* $\chi \in \Pi(\overline{A})_u$ *such that (*$\chi^{w_0} \neq \chi$ *and)* $\pi = I_{\overline{A}}(\chi)$*, as in §3.1.*

[4] This was originally only a conjecture in [18]. An anonymous referee of [18] pointed out that it followed from [23].

- *(Complementary series) There is a* $\chi \in \Pi(\overline{A}) - \Pi(\overline{A})_u$ *such that* $\pi = I_{\overline{A}}(\chi)$*, as in §3.2.*
- *("Reducible principal series") There is a* $\chi \in \Pi(\overline{A})_u$ *such that* ($\chi^{\overline{w_0}} = \chi$ *and)* π *is either a subrepresentation or a quotient of* $I_{\overline{A}}(\chi)$*, as in §3.1.*
- π *is a "special" or "trash" representation as in §3.3.*
- π *is a supercuspidal representation as in [19].*

Theorem 3.14. *Let* $\overline{G}$ *be as in §1.1 above. If* $\pi \in \Pi(\overline{G})_t$ *then one of the following holds.*

- *(Principal series) There is a* $\chi \in \Pi(\overline{A})_u$ *such that* ($\chi^{\overline{w_0}} \neq \chi$ *and)* $\pi = I_{\overline{A}}(\chi)$*, as in §3.1.*
- *("Reducible principal series") There is a* $\chi \in \Pi(\overline{A})_u$ *such that* ($\chi^{\overline{w_0}} = \chi$ *and)* π *is either a subrepresentation or a quotient of* $I_{\overline{A}}(\chi)$*, as in §3.1.*
- π *is a "special" representation as in §3.3.*
- π *is a supercuspidal representation as in [19].*

4 Invariant distributions

We classify the image of $C_c^\infty(\overline{G})$ and of $\mathcal{S}(\overline{G})$ under the "scalar-valued Fourier transform" or "trace map", $f \longmapsto \operatorname{tr}\pi(f)$, where $\overline{G}$ is as in §1.1 above. We prove that all invariant distributions on $\overline{G}$ are supported on tempered characters, where $\overline{G}$ is either a cover of $SL(2, \mathbf{F})$ or a cover of $GL(r, \mathbf{F})$ as in §1.1. Finally, we show that, for $\overline{G}$ as in §1.1, we can write any invariant tempered distribution D on $\overline{G}$ as an integral on the tempered dual.

4.1 Tempered Paley-Wiener theorem

In this section, $\overline{G}$ is a cover of $SL(2, \mathbf{F})$ as in §1.1. Let $C \subset A$ be a maximal subgroup of A for which $\overline{C} \subset \overline{A}$ is abelian.

Next we classify the image of the Fourier transforms of a "generic" *unitary* principal series representation $\pi = I_{\overline{A}}(\chi)$, (where $\chi = Ind_{\overline{C}}^{\overline{A}}\mu \in \Pi(\overline{A}), \mu \in \Pi(\overline{C})$), on $C_c^\infty(\overline{G})$. Note that both the Weyl group W and $H = \overline{A}/\overline{C}$ act on $C_c(\Pi(\overline{C}))$ by conjugation and $\phi_f \in C_c(\Pi(\overline{C}))$. Let $C_c(\Pi(\overline{C}))^H$ denote the subspace of H-invariant functions and let $C_c(\Pi(\overline{C}))^{WH}$ denote the subspace of functions which are both H-invariant and W-invariant.

Recall the Fourier transform with respect to the principal series, $\phi_f(\mu) = \Theta_\pi(f)$, (where $\pi = I_{\overline{A}}(\chi), \chi = Ind_{\overline{C}}^{A}\mu \in \Pi(\overline{A}), \mu \in \Pi(\overline{C})$), When μ is unitary we call this the Fourier transform with respect to the *unitary principal series.* If the restriction of μ to an $diag(x, x^{-1}) \in A^n$ is of the form $|x|^s$ then we write $\phi_f(\mu) = \phi_f(s)$. When μ is of this form and s is real, we call this the Fourier transform with respect to the *complementary series.*

Proposition 4.1. *For $f \in C_c^\infty(\overline{G})$, the image $C_c^\infty(\overline{G})^\wedge_{ps}$ of the Fourier transform $f \longmapsto \phi_f$ with respect to the unitary principal series, is given by*

$$C_c^\infty(\overline{G})^\wedge_{ps} = \left\{ h \in C_c(\Pi(\overline{C})_u)^{WH} \mid \begin{array}{c} h \text{ is a trig polynomial on} \\ \text{each circle in } \Pi(\overline{C})_u \end{array} \right\}.$$

The image $C_c^\infty(\overline{G})^\wedge_{cs}$ of the Fourier transform $f \longmapsto \phi_f$ with respect to the complementary series, is given by

$$C_c^\infty(\overline{G})^\wedge_{cs} \cong \left\{ h \in C_c^\infty(\Pi(\overline{C}))^{WH} \text{ restricted to } 0 < s < 1/n, \text{ a polynomial in } q^s \right\}.$$

This follows from character formulas for induced representations and from results on p-adic Mellin transforms in [29], pp. 43-44. Further details may be found in a slightly expanded version of this paper [20].

Analogous to Proposition 4.1 above, we have the following result.

Proposition 4.2. *For $f \in \mathcal{S}(\overline{G})$, the image $\mathcal{S}(\overline{G})^\wedge_{ps}$ of the Fourier transform $f \longmapsto \phi_f$ with respect to the unitary principal series, is given by*

$$\mathcal{S}(\overline{G})^\wedge_{ps} = C_c^\infty(\Pi(\overline{C})_u)^{WH}.$$

4.2 The Fourier transform

Let

$$J = \text{span}\left\{ f - f^g \mid f \in C_c^\infty(\overline{G}) \,, g \in \overline{G} \right\},$$

and recall $PW_t(\overline{G})$ denotes the tempered Paley-Wiener space.

Proposition 4.3. *Let $\overline{G}$ be as in §1.1. The kernel of the trace map*

$$\wedge : C_c^\infty(\overline{G}) \to PW_t(\overline{G}),$$

defined by $f^\wedge(\pi) = \Theta_\pi(f)$, is J.

Proof. For $\overline{G}$ as in §1.1, Vignéras (see Proposition 3.2 and §2.3 in [30]) showed that the kernel of the orbital integral map $\Phi : C_c^\infty(\overline{G}_r) \to C_c^\infty(\overline{G}_r)^{\overline{G}}$ is J. Let K denote the kernel of the trace map $^\wedge : C_c^\infty(\overline{G}) \to PW_t(\overline{G})$. The Weyl integration formula implies $J \subset K$. Theorem 19.2 of Flicker and Kazhdan [9] [5] implies $K \subset J$, if $\overline{G}$ is a metaplectic cover of $GL(r, \mathbf{F})$. If $\overline{G}$ is a cover of $SL(2, \mathbf{F})$ as in §1.1 above then Fourier transforms of Harish-Chandra transforms and character formulae for induced representations (implicit in §3.1 above but see also explicit formulas in [18] or [16], for example) show that if $f \in K$ then $F_f^{A^N}(a) = 0$. It remains to show that if $f \in K$ then $F_f^T(t) = 0$, where $T = Cent(t, \overline{G})$, is the centralizer of a regular elliptic element (see (5) above). The desired $F_f^T(t) = 0$ follows from (3.43) in [15]. □

Let V' denote the dual of the complex vector space V. If V is in addition a $\overline{G}$-module, let $(V')^{\overline{G}}$ denote the subspace of $\overline{G}$-invariant linear functionals.

Lemma 4.4. *Let $\overline{G}$ be as in §1.1. The canonical map*

$$(C_c^\infty(\overline{G})/J)' \to (C_c^\infty(\overline{G})')^{\overline{G}}$$

is an isomorphism.

Proof. For $f \in C_c^\infty(\overline{G})$, let $f \bmod J$ denotes its class in $C_c^\infty(\overline{G})/J$. First, note that the canonical map

$$\begin{gathered} (C_c^\infty(\overline{G})/J)' \to (C_c^\infty(\overline{G})')^{\overline{G}} \\ D \longmapsto D^* \\ (f \bmod J \longmapsto D(f \bmod J)) \longmapsto (f \longmapsto D(f \bmod J)). \end{gathered}$$

is injective by definition.

To see that this is surjective, let $D \in (C_c^\infty(\overline{G})')^{\overline{G}}$. We must show that there is a $D_0 \in (C_c^\infty(\overline{G})/J)'$ such that $D = D_0^*$. Let

$$D_0(f \bmod J) = D(f), \qquad f \in C_c^\infty(\overline{G}).$$

We want to show that D_0 is a well-defined distribution, i.e., that if $f, f' \in C_c^\infty(\overline{G})$ and $f \bmod J = f' \bmod J$ then $D(f) = D(f')$. By definition of J, $f \bmod J = f' \bmod J$ implies $f' = f + \sum_{i \in I} c_i(f_i - f_i^{g_i})$, for some finite set I and some $c_i \in \mathbb{C}$, $f_i \in C_c^\infty(\overline{G})$, $g_i \in \overline{G}$. Since D is invariant, from linearity it follows that $D(f) = D(f')$, as desired. Therefore, the canonical map is surjective. □

From these two results, we conclude the following important fact.

[5] This section of [9] uses the global trace formula, hence requires the assumption that n is relatively prime to all composite positive integers less than or equal to r and to the residual characteristic over $\mathbf{F}$.

Theorem 4.5. *Let $\overline{G}$ be as in §1.1 above. The trace map*

$$\wedge : C_c^\infty(\overline{G}) \to PW_t(\overline{G})$$

factors through the canonical map

$$(C_c^\infty(\overline{G})/J)' \to (C_c^\infty(\overline{G})')^{\overline{G}}.$$

In other words, each invariant distribution is supported on tempered characters.

This result allows us to define, for each invariant distribution D on G, the *Fourier transform* $D^\wedge$ on $PW_t(\overline{G})$ by

$$D(f) = D^\wedge(f^\wedge), \qquad f \in C_c^\infty(\overline{G}). \tag{7}$$

By the results of §3 above and of [9], the tempered dual has both a continuous part and a discrete part. The continuous part of $PW_t(\overline{G})$ decomposes into a vector space sum of smooth functions on compact real tori. It is noted for later reference that if $D^\wedge$ induced, by restriction, a distribution on each of these spaces of smooth functions then D must be tempered.

4.3 The Fourier transform as an integral over $\Pi(\overline{G})_t$

Note that any function m on the tempered dual $\Pi(\overline{G})_t$ extends by linearity to the Grothendieck group $R(\overline{G})_t$ (defined in [22] in the algebraic case; in the metaplectic case the definition is similar).

In the case $n = 1$, let $d\omega$ denote the canonical measure on the discrete dual of G as in §2 of [14]. The discrete dual has the structure of the disjoint union of compact real manifolds $\mathcal{O}$. Using Corollary 4.5.11 and Theorem 4.6.1 in [28], we may extend this measure to $\Pi(G)_t$, which is parameterized by (a dense subset of) the discrete dual.

In case $G = GL(r, \mathbf{F})$, we use the correspondence between $\Pi(G)_t$ and $\Pi(\overline{G})_t$ proven in §19 of [9] to pull these parameters and measures on $\Pi(G)_t$ back to $\Pi(\overline{G})_t$. In case $G = SL(2, \mathbf{F})$, we use the correspondence between $\Pi(G)_t$ and $\Pi(\overline{G})_t$ proven above to pull these parameters and measures on $\Pi(G)_t$ back to $\Pi(\overline{G})_t$. Let $d\mu$ denote the measure on the tempered dual $\Pi(\overline{G})_t$ corresponding to $d\omega$. Let $m(\pi)d\mu(\pi)$ denote a distribution on the tempered Paley-Wiener space $PW_t(\overline{G})$ such that

1. $m(\pi)$ is supported on finitely many orbits $\mathcal{O} = \mathcal{O}_\sigma$, for a genuine discrete series representation σ of some Levi M of $\overline{G}$,

2. there is a continuous function h on $\Pi(\overline{G})_t$ such that on each orbit $\mathcal{O} = \mathcal{O}_\sigma$, with σ, M as above, such that as distributions on $C_c^\infty(\mathcal{O})$, we have

$$m(I_M(\omega\sigma)) = \frac{\partial^I}{\partial\omega^I} h(\omega),$$

where $I = (i_1, ..., i_r)$ denotes a multi-index, r being the real dimension of $\mathcal{O}$, and $\frac{\partial^I}{\partial\omega^I}$ denotes partial differentiation on the real manifold $\mathcal{O}$.

We call such a distribution a *distribution of finite type* on $\Pi(\overline{G})_t$. A distribution satisfying (2) but not (1) will be called a *distribution of quasi-finite type* on $\Pi(\overline{G})_t$. The maximum of the integers $|I| = i_1 + ... + i_r$, where I runs over all multi-indices occurring in (2), is called the *order* of the distribution.

Theorem 4.6. *Let $\overline{G}$ be as in §1.1 above. If D is an invariant tempered distribution on $C_c^\infty(\overline{G})$ then there is a distribution of quasi-finite type $m(\pi)d\mu(\pi)$ on the tempered Paley-Wiener space $PW_t(\overline{G})$ such that*

$$D(f) = \int_{\Pi(\overline{G})_t} \Theta_\pi(f) m(\pi) d\mu(\pi), \qquad f \in C_c^\infty(\overline{G}).$$

This formula extends continuously to all of $\mathcal{S}(G)$.

Remark 4.7. *If we replace $\mathcal{S}(\overline{G})$ by $\mathcal{S}_K(\overline{G})$ in the second part of the above theorem then we can replace quasi-finite by finite.*

Proof. First, we know from Theorem 4.5 that D is supported on the tempered dual.

We claim that the tempered dual is contained in the unitary dual. In the $SL(2)$ case, see [16] for the detailed case-by-case proof using the classification of the irreducible admissible representations of $\overline{G}$. In the $GL(r)$ case, see §§16-17 of Flicker-Kazhdan [9] [6]. Therefore D arises from a distribution $D^\wedge$ on $\Pi(\overline{G})_t$. The above theorem is now an immediate consequence of equation (7), and the classification of L. Schwartz ([27], ch. III, Th. XXI) which we state in the present notation as follows.

Lemma 4.8. *(Schwartz) Let $\mathcal{O}$ be an orbit as above. If $T \in C_c^\infty(\mathcal{O})'$ then there is a continuous function h on $\mathcal{O}$ and a multi-index $I = (i_1, ..., i_m)$, $i_j \geq 0$ such that $T = \frac{\partial^I}{dx^I} h$ (as distributions), where $x = (x_1, ..., x_m)$ is a coordinate on $\mathcal{O}$.*

This completes the proof of the Theorem. □

[6] These sections of [9] do not use the global trace formula, hence only requires the assumption that n is relatively prime to the residual characteristic of $\mathbf{F}$.

4.4 Some corollaries

By Theorem 4.5, the Fourier transform of each invariant distribution is support on the tempered dual.

Corollary 4.9. *Let $\overline{G}$ be as in §1.1 above. If D is tempered then the Fourier transform $D^\wedge$ may be expressed in the form*

$$D^\wedge(h) = \int_{\Pi(\overline{G})_t} h(\pi)m(\pi)d\mu(\pi),$$

for all $h \in PW_t(\overline{G})$, where $m(\pi)d\mu(\pi)$ is a quasi-finite distribution.

It is natural to ask for a more explicit characterization of the admissible distributions [13]. The result below uses the above corollary to basically reduce the question of admissibility down to the behaviour of the distribution near the singular set.

Corollary 4.10. *Assume $\overline{G} = G$ ($n = 1$). If D is an tempered then it is admissible on the regular set. In other words, if we identify $C_c^\infty(G_r)$ with the following subspace of $C_c^\infty(G)$,*

$$C_c^\infty(G_r) = \{f : G \to \mathbb{C} \mid f|_{G-G_r} = 0,\ f \in C_c^\infty(G_r)\},$$

then $D|_{C_c^\infty(G_r)}$ is admissible.

The proof may be found in a slightly expanded version of this paper [20].

Example 4.11. *Clearly $f \longmapsto f(1)$ is a tempered distribution. Theorem 4.6 implies that there is a quasi-finite m such that*

$$f(1) = \int_{\Pi(\overline{G})_t} \Theta_\pi(f)m(\pi)d\mu(\pi), \qquad f \in C_c^\infty(\overline{G}).$$

This is a weak case of Harish-Chandra's Plancherel theorem.

References

[1] Ariturk, On the composition series of principal series representations of a three-fold covering group of $SL(2, K)$, *Nagoya Math J.* **77**(1980)177-196

[2] J. Arthur, The trace formula in invariant form, *Ann. Math.* **114**(1981)1-74

[3] ——, The trace Paley-Wiener theorem for Schwartz functions, *Contemp. Math.* 177(1994)171-180

[4] J. Benedetto, *Spectral Synthesis*, Academic Press, Inc., 1975

[5] J. Bernstein, Le centre de Bernstein , (notes by P. Deligne), in *Représentations des groupes réductifs sur un corp local*, Hermann, Paris, 1984

[6] —– and A. Zelevinski, Representations of the group $GL(n, F)$, where F is a non-archimedian local field, *Russian Math Surveys* **31**(1976)1-68

[7] —– and A. Zelevinski,, Induced representations of reductive p-adic groups, I, *Ann. scient. Éc. norm. Sup.* **10**(1977)441-472

[8] C. Davis, keynote talk presented at the Conference on Harmonic Analysis in honor of John Benedetto, University of Maryland, October, 1999.

[9] Y. Flicker and D. Kazhdan, Metaplectic correspondence, *Publ. Math. IHES* **64**(1986)53-110

[10] S. Gelbart, *Weil's Representation and the Spectrum of the Metaplectic Group*, Springer Lecture Notes in Math **530**, 1976

[11] —– and P. Sally, Intertwining operators and automorphic froms on the metaplectic group, *Proc. Nat. Acad. Sci.* **72**(1975)1406-1410

[12] Harish-Chandra, *Harmonic analysis on reductive p-adic groups*, (notes by G. van Dijk), Springer Lecture Notes in Math **162**, 1970

[13] ——, Admissible invariant distributions on reductive p-adic groups, *Queen's papers in pure and applied math.* 48(1978) 281-347

[14] ——, The Plancherel formula for reductive p-adic groups , in *Collected Works*, vol IV, ed. V.S. Varadarajan, Springer-Verlag, New York, 1984 (see also the paper Corrections ..., in the same volume)

[15] D. Joyner, On the metaplectic analog of Kazhdan's 'endoscopic' lifting, *Isr. J. Math.* 61 (1988)113-154

[16] ——, A hitch-hiker's guide to genuine invariant distributions on metaplectic covers of SL(2) over a p-adic field, preprint, 1995 (available from `http://web.usna.navy.mil/~wdj/papers.html`)

[17] ——, A correspondence for the generalized Hecke algebra of the metaplectic cover $\overline{SL(2, F)}$, F p-adic, *New York J. of Math.* 4 (1998) 223-235.

[18] ——, On the unitary dual of $\overline{SL(2, F)}$, F p-adic, preprint, 1999 (available from `http://web.usna.navy.mil/~wdj/papers.html`)

[19] ——, A correspondence for the supercuspidal representations of $\overline{SL_2(F)}$, F p-adic, *Archiv der Math.* 273(1999)1-9

[20] ——, Invariant distributions on the n-fold metaplectic covers of $GL(r, F)$, F p-adic, preprint, 2000 (a more detailed version of this paper, available from `http://web.usna.navy.mil/~wdj/papers.html`)

[21] — and Courtney Moen, The trace formula for $\overline{SL(2, \mathbb{A})}$ and applications , preprint, 2000.

[22] D. Kazhdan, Cuspidal geometry of p-adic groups, *J. d'Anal. Math.* 47(1986)1-36

[23] — and S. Patterson, Metaplectic forms, *Publ. Math. IHES* **59**(1984)35-142

[24] P. Mezo, A global comparison for general linear groups and their metaplectic coverings, Univ. of Toronto PhD thesis (advisor: James Arthur), 1998

[25] C. Moen, Intertwining operators for covering groups of $SL(2)$ over a local field, preprint

[26] ——, Irreducibiliy of unitary principal series representations for covering groups of $SL(2)$, *Pac. J. Math.* 135(1988)89-110

[27] L. Schwartz, *Theorie des distributions, vol. I*, Hermann, Paris, 1950

[28] A. Silberger, *Introduction to harmonic analysis on reductive p-adic groups*, Princeton Univ. Press, Princeton, 1979

[29] M. Taibleson, *Fourier analysis on local fields*, Princeton Univ. Press, Princeton, 1975

[30] M.-F. Vignéras, Caractérisation des integrales orbitales sur un groupe réductif p-adique, *J. Fac. Sci. Univ. Tokyo* **28**(1981)945-961

[31] A. Weil, *Basic Number Theory*, Springer-Verlag, 1975

David Joyner
Mathematics Department
US Naval Academy
Annapolis, Maryland
USA
email: wdj@usna.edu